Nonimpulsively Started Steady Flow About a Circular Cylinder

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The effect of constant acceleration, prior to the establishment of a steady uniform flow, on some of the characteristics of the resulting time-dependent flow about a circular cylinder is investigated experimentally. It is shown that the occurrence of a local maximum drag, the onset of wake asymmetry, and the evolution of the transverse force are dependent on the parameters characterizing the nonimpulsive nature of the ambient flow.

Nomenclature

A_p = acceleration parameter, $1/(S/R)_v$
A_{pn} = $D^n(d^nU/dt^n)/V^{n+1}$
C_d = drag coefficient, $2F/\rho DV^2$
C_{dm} = maximum drag coefficient
C_i = $4F/(\pi\rho D^2\, dU/dt)$
C_L = lift coefficient, $2L/\rho DV^2$
D = diameter of circular cylinder, $2R$
F = drag or in-line force per unit length
L = lift or transverse force per unit length
R = radius of the circular cylinder
Re = VD/ν, Reynolds number
S = displacement of fluid
S/R = relative displacement of fluid
$(S/R)_m$ = relative fluid displacement at which C_{dm} occurs
$(S/R)_v$ = relative displacement of fluid during the acceleration period, $=0.5(dU/dt)t_v^2/R = V^2/(D\,dU/dt) = 1/A_p = 0.5(Vt_v/R)$
St = Strouhal number, D/VT
T = period of vortex shedding
t = time
t_v = time at the end of the acceleration period
U = time-dependent velocity
V = constant velocity at the end of the acceleration period

Introduction

UNSTEADY flow past bluff bodies has attracted a great deal of attention since a number of problems of practical importance are unsteady. Among the numerous theoretical, numerical, and experimental investigations, "impulsively started" steady flow about a circular cylinder has occupied a prominent place partly because of its intrinsic interest towards the understanding of the evolution of separation, vortex formation, and growth; partly because of its practical importance in various aerodynamic applications (e.g., the impulsive flow analogy, flow about missiles, dynamic stall); and partly because it provided the most fundamental case for the comparison and validation of various numerical methods and codes. However, neither impulsive start nor impulsive stop is physically reliable. The flow must be accelerated from rest to a constant velocity or decelerated from a constant velocity to rest, or to another velocity, in a prescribed manner. This fact gives rise to a series of new questions: 1) What is the effect of the initial acceleration, prior to the establishment of a steady uniform flow, on the characteristics of the resulting time-dependent flow? 2) Are there critical values of the governing parameters above or below which the flow may be regarded as almost impulsively started? 3) How does the rate of accumulation of vorticity, as well as its cross-wake transfer, depend on the initial history of the motion? The purpose of this investigation is to explore some of these questions through the use of a constant, rather than arbitrary, acceleration of the ambient flow over a specified period.

Background Studies

There is a large volume of literature that deals with fluctuating forces and associated vortex shedding from bluff bodies subjected to steady ambient flow. The circular cylinder has attracted by far the greatest attention. Numerous computational studies have been performed on flow about circular cylinders in an attempt to predict the Strouhal number in steady ambient flow and the shape and growth of the wake region in impulsively started steady flow. Here only the more recent and relatively more accurate examples will be cited. Loc[1] solved the complete unsteady Navier-Stokes equations in vorticity/stream function form using a combination of second- and fourth-order compact finite-difference schemes. He obtained short-time symmetric-wake solutions at Reynolds numbers of 300, 550, and 1000 and achieved good agreement with flow visualization results for both vortex size and center position. His calculations also showed clearly the small secondary vortices just behind the separation points.

Lecointe and Piquet[2] used several compact schemes with the Navier-Stokes vorticity/stream function formulation to solve laminar flows around circular cylinders up to a Reynolds number of 9500. They studied both startup and unsteady periodic phenomena. The predicted wake region shape showed good agreement with experimental flow visualizations. Loc and Bouard[3] performed calculations at $Re = 3000$ and 9500 using a fourth-order finite-difference technique for the Poisson equation for the stream function and a second-order technique for the vorticity-transport equation. They found good agreement between their predictions and flow visualization. The calculations were confined, out of necessity, to relatively short times during which the wake became neither asymmetrical nor turbulent. Chamberlain[4] used a second-order fast Poisson solver based on FFT methods and found an accurate solution that agreed well with experiment and the previous computations. Rumsey[5] used an upwind-biased implicit approximate factorization algorithm to calculate the impulsively started unsteady flow over a circular cylinder at a Reynolds number of 1200 and a Mach number of 0.3. Rumsey's results were in very good agreement with the previous calculations and showed, predictably enough, only a slight compressibility effect.

All numerical calculations using finite-difference, finite element, or vortex-element methods (see, e.g., Sarpkaya and

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Shoaff,[6] van der Vegt,[7] Sarpkaya,[8], and Chang and Chern[9]) have assumed an impulsively started flow. No computational attempt was made to investigate the effect of the initial acceleration, prior to the establishment of a steady uniform flow, on the characteristics of the resulting time-dependent flow.

Several experimental investigations[10-12] of impulsively started flow around circular and rectangular cylinders have been carried out. Bouard and Coutançeau[10] investigated the shape and growth rate of the wake region behind the cylinder for Reynolds numbers between 40 and 10,000. Sarpkaya[11-12] examined the evolution of the wake region and the development of the lift and drag forces with time for cylinders for Reynolds numbers between 15,000 and 120,000. Nagata et al.[13] studied the startup flow at Reynolds numbers between 250 and 1200, with the majority of the experiments performed at $Re = 1200$. They gave detailed results for the time evolution of the vortical region, boundary-layer parameters, and profile shapes at this Reynolds number. Sarpkaya and Kline[14] examined the impulsively started flow about four types of bluff bodies. Sarpkaya and Ihrig[15] performed experiments and vortex-element analysis of impulsively started flow about rectangular prisms and pointed out emphatically that, other than numerical experiments, there is no mechanical system that is capable of generating a truly impulsive flow. In fact, efforts to generate impulsive or uniformly accelerated flow at high Reynolds numbers may be hampered by the generation of compression and rarefaction waves and regions of intense cavitation (in liquids). For this reason one or more acceleration parameters such as

$$A_p = D\frac{dU}{dt}V^2 \tag{1a}$$

or

$$A_{pn} = D^n\frac{d^nU}{dt^n}V^{n+1} \tag{1b}$$

will have to be added to the list of the parameters governing the phenomenon in order to account for the initial history of the fluid motion. The other parameters are $Re = VD/\nu$ and the relative displacement of the ambient flow, given by

$$S/R = 0.5\frac{dU}{dt}t^2/R = \frac{0.5Vt^2}{Rt_v} \qquad \text{for } t \le t_v \tag{2a}$$

and

$$S/R = 0.5\left(\frac{Vt_v}{R}\right) + (t - t_v)V/R \qquad \text{for } t > t_v \tag{2b}$$

where U is the time-dependent velocity in the interval $(0 < t < t_v)$. A systematic experimental variation of such parameters for an arbitrary $U(t)$ is extremely difficult. Thus, to make progress, one must begin with the simplest unsteadiness, namely, with constant dU/dt, so as to be able to incorporate progressively more complex variations of velocity with time.

Experiments

Experiments were conducted in a 17-ft-high (5.2-m), 2×2 ft (0.61×0.61 m) cross section vertical water tunnel. A computer-controlled quick-release valve, located at the base of the tunnel, is used to create impulsively or nonimpulsively started flow of desired velocity history (for additional details see Sarpkaya[12] and Kline[16]). Two circular cylinders of diameters 2.5 in. (6.35 cm) and 2.95 in. (7.5 cm) were used in the experiments. Each end of the model was extended into a 1-in. (25.4-mm) deep cylindrical cavity, in each side wall, to minimize end effects. The diameter of the cavity was approximately 0.04 in. (1 mm) larger than the cylinder diameter to allow unrestrained transmission of the lift and drag forces to the load gages. Two extension rods of 1 in. (25.4 mm) diam were attached rigidly to the ends of the model. Then they were inserted into the self-aligning ball bearings of the strain-gaged load cells, mounted on the outer faces of the tunnel windows.[12-16] The gages had ample dynamic response for the measurements reported herein.

The velocity of the terminal flow ranged from 0.75 ft/s to 2 ft/s. It was measured through the use of a variable-resistance probe. A 13-ft- (4 m) long platinum wire, stretched vertically in the tunnel and mounted away from the walls, provided water level indication to a data acquisition system. The response of the wire was perfectly linear within the range of fluid displacements encountered. The ambient velocity was also determined from the integration of the instantaneous acceleration. The Reynolds number ranged from about 16,000 to 50,000.

The instantaneous acceleration was measured by means of a differential pressure transducer connected to two pressure taps, placed two ft apart in the direction of the ambient flow. The instantaneous acceleration was then calculated from $\Delta p = \rho\Delta\ dU/dt$, where Δp is the differential pressure, ΔH is the distance between the pressure taps, and dU/dt is the instantaneous acceleration of the fluid. There was no difficulty in imparting a large rate of change of initial acceleration to the fluid in arriving at a constant acceleration in a very short time (normally in about 0.05 s with $d^2U/dt^2 = 60$–100 ft/s^3). In other words, $D^2(d^2U/d^2)/V^3$ did not measurably affect the subsequent flow, and the effects of the initial rate of change of acceleration were obscured by the subsequent acceleration effects quantified by $A_p = D(dU/dt)/V^2$. However, the shape of the transition from the constant-acceleration phase to the constant-velocity phase (at the end of the acceleration period) turned out to be one of the most important parameters in determining the magnitude and position of the drag overshoot. In fact, the manner the acceleration is removed from the flow to sustain a constant velocity turned out to be more important than the manner in which the acceleration is initially imposed on the flow. Special valve-control mechanisms had to be developed in order to ensure that the transition from constant acceleration to constant velocity V was as rapid as possible, i.e., $|d^2U/dt^2|$ was as large as possible, and that there was no residual unsteadiness in the early stages of the constant-velocity phase. This realization was brought about by a careful re-examination of the data taken about 25 years ago in another vertical water tunnel by Sarpkaya.[11] It was discovered that the reason for the occurrence of the maximum drag at an S/R value (near $S/R \approx 8$) larger than that encountered by Sarpkaya[12] in 1978 and in the present investigation was neither the rate nor the duration of the initial acceleration but rather the presence of a small residual deceleration during the initial phases of the constant velocity (spanning over a relative displacement of about $S/R \approx 5$). This was caused by the unsteady flow separation about the blades of the butterfly valve, controlling the acceleration. The consequences of this residual deceleration was not fully appreciated 25 years ago, primarily because of the repeatability of the velocity and force histories. A new tunnel was constructed in 1978, and the butterfly valve was replaced by an axisymmetric quick-opening valve, specially tailored and tightly controlled by means of a computer to make the transition from constant acceleration to constant velocity as sharp as possible[12]; i.e., with the largest $|d^2U/dt^2|$ as possible (50–100 ft/s^3). The acceleration system has been further improved since then to carry out the experiments described herein.

The acceleration parameter A_p was varied from about 0.05 to 1.2, and $D^2(d^2U/dt^2)/V^3$ was varied from about 0.5 to 5. Before any experiments were conducted, impulsive flow was initiated several times to check the operation of the system. Adjustments to the control system of the quick-release valve were made, as necessary, to ensure the repeatability of the desired variation in velocity and acceleration.

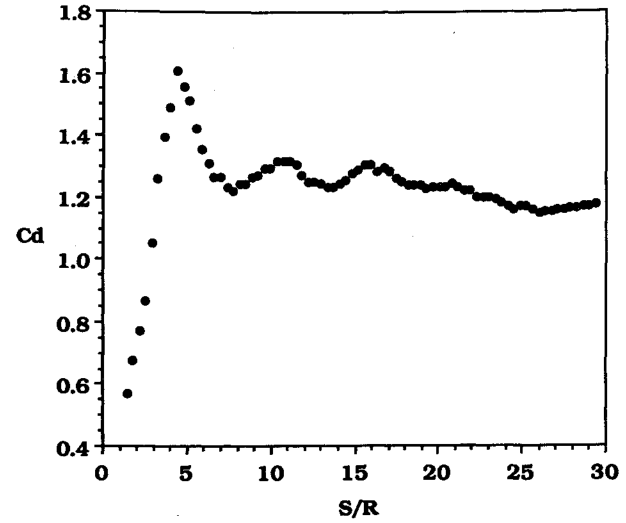

Fig. 1a Drag coefficient vs relative displacement for $A_p = 1.18$.

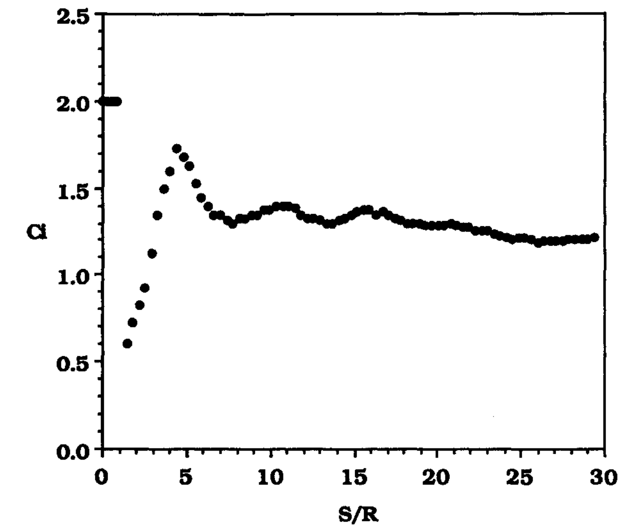

Fig. 1b Force coefficient C_i vs relative displacement for $A_p = 1.18$.

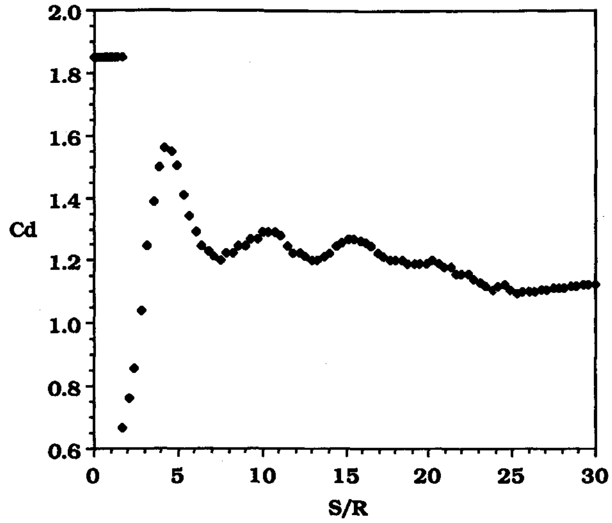

Fig. 2a Drag coefficient vs relative displacement for $A_p = 0.59$.

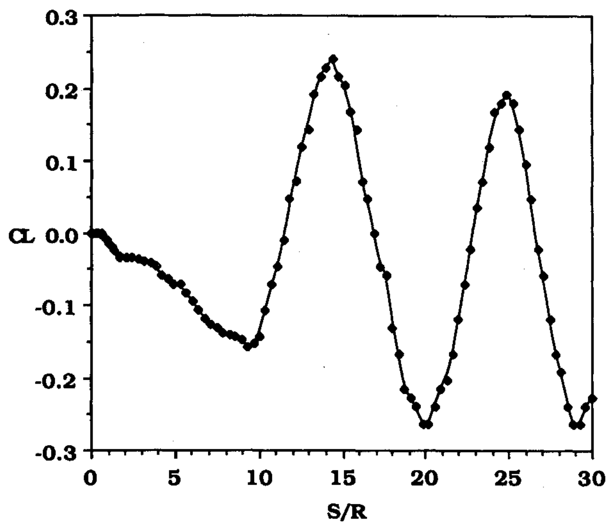

Fig. 2b Lift coefficient vs relative displacement for $A_p = 0.59$.

Results

The drag force F was normalized either by $(\rho D V^2/2)$ to calculate $C_d = 2F/(\rho D V^2)$ or by $[(\pi \rho D^2 \, dU/dt)/4]$ to calculate $C_i = 4F/(\pi \rho D^2 \, dU/dt)$ from the data as a function of the relative displacement of the fluid, S/R [see Eqs. (2a) and (2b)], for various values of A_p. Note that the ratio of the two coefficients yields $C_d/C_i = (\pi/2)A_p$ for all values of S/R.

Figure 1a shows C_d for $A_p = 1.18$ and $Re = 46{,}200$. At the start of the motion and prior to separation, the flow is essentially potential in nature, and the drag is basically due to inertial forces. For a circular cylinder the force coefficient C_i has a potential-flow value of 2 (due to added mass plus the pressure gradient to accelerate the ambient flow, see, e.g., Sarpkaya and Isaacson[17]). Thus, near $t = 0$, the initial value of C_d, denoted by C_{din}, is equal to πA_p. For the data shown in Fig. 1, $C_{din} = 3.7$ (beyond the range of the vertical scale). Figure 1b is a replot of the data shown in Fig. 1a in terms of C_i. Clearly, the initial value of C_i is nearly equal to its theoretical value of 2 at the start of the acceleration (see data points near $C_i = 2$ for S/R smaller than about 0.85). As soon as the acceleration is removed at $(S/R)_v = V^2/(D \, dU/dt) = 1/A_p \approx 0.85$, C_i drops sharply to about 0.3, as expected on the basis of $C_d/C_i = (\pi/2)A_p$, with $C_d = 0.56$ from Fig. 1a. The remainder of Fig. 1b does not imply that the acceleration-dependent inertial force and the velocity-square-dependent drag force can be separated easily even if the total force were to be assumed to have comprised of two such simple parts. A detailed discussion of the complex problems involved in the description of the drag and inertial forces in time-dependent flows is given by Sarpkaya and Isaacson.[17] Figure 1a shows that C_d increases rapidly to a value of about $C_{dm} = 1.6$ at $(S/R)_m = 4.5$, decreases just as rapidly to a value of about 1.2, and then exhibits two rather large fluctuations before settling down to a mean value of about 1.15. The difference between $(S/R)_m$, at which the maximum C_d occurs, and $(S/R)_v$, at which the acceleration period ends, is $\Delta(S/R) = (4.5 - 0.85) = 3.65$, i.e., the flow is in the constant-velocity regime for a relative distance of 3.65 before C_d exhibits an overshoot. Sarpkaya's[12] previous data at $A_p = 1$ and $Re \approx 20{,}000$ and 32,000 gave similar levels of C_{dm}, $(S/R)_m$, and $\Delta(S/R)$.

Figures 2a and 2b show the drag and lift coefficients for $A_p = 0.59$ and $Re \approx 46{,}000$. The experimental value of $C_{din} = 1.85$ is in agreement with that given by $C_{din} = \pi A_p$, showing that the initial value of $C_i \approx 2$ during the entire acceleration period, i.e., for $(S/R)_v < A_p^{-1} = 1.69$. Also noted is the fact that the drag overshoot occurs, as before, at $S/R \approx 4.5$ even though $\Delta(S/R)$ is now $(4.5 - 1.69) = 2.81$. The drag data shown in Figs. 1a and 2a are repeatable within $\pm 5\%$.

The lift coefficient shown in Fig. 2b is representative of many such figures. Depending on the unknown initial distur-

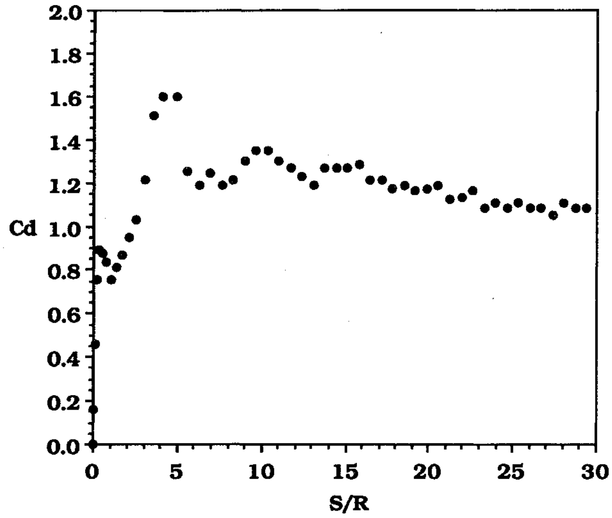

Fig. 3a Drag coefficient vs relative displacement for $A_p = 0.27$.

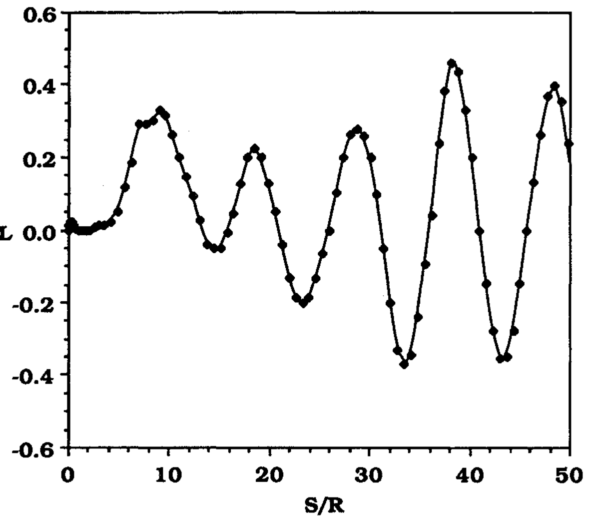

Fig. 3b Lift coefficient vs relative displacement for $A_p = 0.27$.

bances, the evolution of the lift force can and does exhibit wide variations, as demonstrated in Sarpkaya.[12] The important facts to be noted in any run are that 1) at the time of drag overshoot, C_L is negligibly small (indicating that the vortex pair is still nearly symmetrical); 2) the first reversal in the lift force occurs near $S/R = 9-10$, and 3) in most cases the period of lift oscillations decreases with increasing S/R. From Fig. 2b $\Delta(S/R) = 10.8$ between the first two minima, corresponding to $St = 0.185$, and $\Delta(S/R) = 9.2$ between the second and third minima, corresponding to $St = 0.217$. This is in conformity with the well-known fact that the elongation of the wake formation region is closely linked to the reduction in shedding frequency.

Figures 3a and 3b show C_d and C_L for $A_p = 0.27$ for which $(S/R)_v = 3.7$. The initial value of $C_{d\text{in}}$ is about 0.85. Thus, from $C_d/C_i = (\pi/2)A_p$, one has $C_i \approx 2$ at the start of motion. The drag overshoot occurs near $S/R \approx 4.5$, followed, as before, by two major oscillations, the first of which corresponds to the shedding of the very first vortex from one side of the cylinder at $S/R \approx 9.5$ and the second to the shedding of the next vortex from the other side of the cylinder at $S/R \approx 15$ (see Fig. 3b). It is rather surprising that the drag overshoot continues to occur at or near $S/R \approx 5$ even though $\Delta(S/R)$ between the drag overshoot and the end of the acceleration period is made smaller (0.8 in Fig. 3a). In fact, a comparison of Figs. 2a ($A_p = 0.59$) and 3a ($A_p = 0.27$) as in Fig. 4 shows that the evolution of C_d with S/R is remarkably similar, except

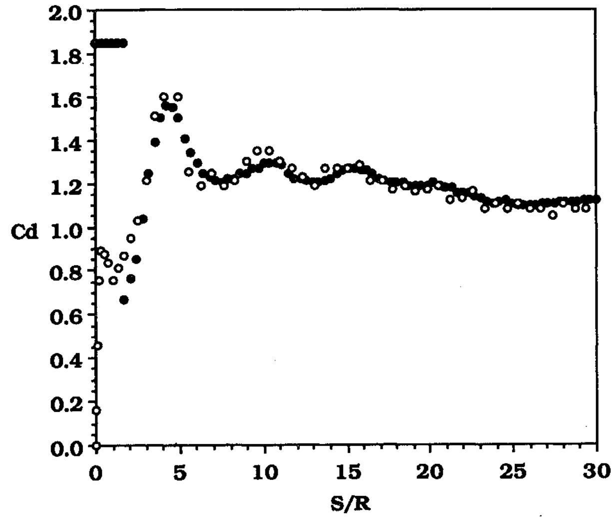

Fig. 4 Comparison of drag coefficients vs relative displacement for $A_p = 0.59$ (full circles) and for $A_p = 0.27$ (open circles).

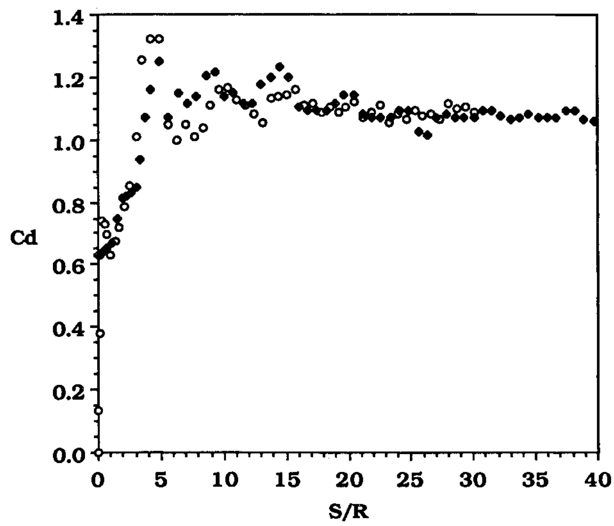

Fig. 5a Drag coefficient vs relative displacement for $A_p = 0.238$ (open circles) and for $A_p = 0.204$ (full circles).

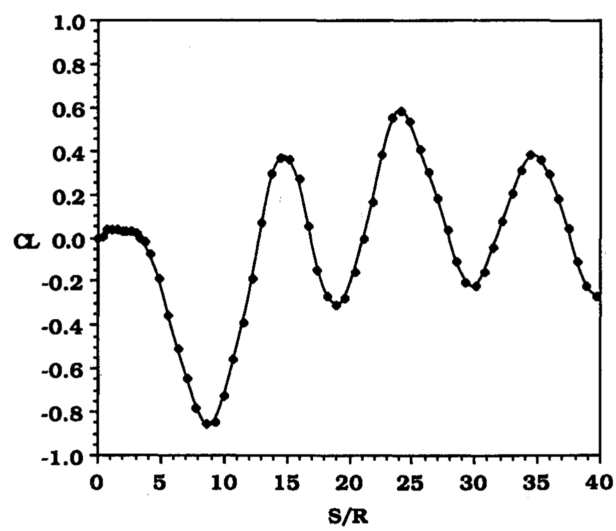

Fig. 5b Lift coefficient vs relative displacement for $A_p = 0.204$.

for the initial value of C_{din} during their respective acceleration periods.

Figure 5a shows C_d for $A_p = 0.204$ [corresponding to $(S/R)_v = 4.9$] and for $A_p = 0.238$ [corresponding to $(S/R)_v = 4.2$], all for $Re \approx 46{,}000$. The case of $A_p = 0.204$ is particularly important because it corresponds to a special $(S/R)_v$ value at which $(S/R)_v \approx (S/R)_m$, the relative displacement at which the local maximum drag normally occurs for all values of A_p larger than 0.204. Thus, the difference in C_d values, say, for $A_p = 1.18$ (Fig. 1a), and $A_p = 0.204$ is a relative measure of the difference between the use of an almost impulsively started steady flow and a uniformly accelerated flow to cause a relative displacement of $(S/R)_v = 4.9$. The second case (with $A_p = 0.238$) corresponds to an intermediate A_p between $A_p = 0.27$ (Fig. 3a) and $A_p = 0.204$. In both cases the drag coefficient increases almost linearly for S/R larger than about 1.5 and reaches a maximum value of about 1.3 at the end of the acceleration period. Then it drops sharply as soon as the acceleration is removed with a large $|d^2U/dt^2|$. As noted earlier, the initial drop is followed by two oscillations that correspond to the shedding of the first two vortices, as seen in Fig. 5b. The third and fourth vortices cause oscillations of smaller amplitude in C_d, as seen in Fig. 5a. The important difference between the cases of $A_p = 0.27$ (Figs. 3a and 4) and $A_p = 0.238$ (Fig. 5a) is that the magnitude of the drag overshoot at or near $S/R \approx 5$ is somewhat smaller (1.3 vs 1.6).

Figures 6a and 6b show C_d and C_i for $A_p = 0.10$ and 0.11, and Fig. 6c shows C_L for $A_p = 0.11$, all for $Re = 46{,}000$. The end of the acceleration period is now at $(S/R)_v = 9.1$ and 10 for $A_p = 0.11$ and 0.10, respectively. The drag coefficient (as well C_i) remains nearly constant for S/R smaller than about 3, increases almost linearly (save for an inflection point to be discussed later) up to the end of the acceleration period, drops rapidly to its commonly accepted quasisteady value as soon as the acceleration is removed, and then undergoes the typical oscillations induced by the shedding of the first few vortices (see Fig. 6c). There is no drag overshoot at $S/R \approx 5$. The behavior of the lift coefficient is also different from the previous cases in several ways. The first vortex sheds at a smaller S/R value (at 7). The second vortex is often very large and sheds after a relative distance of $\Delta(S/R) \approx 4$, which corresponds to $St = 0.25$. However, the Strouhal number based on the first two minima is about 0.20. The data shown in Figs. 6a–6c are typical of the data obtained with A_p values smaller than about 0.16.

Discussion of Results

The data presented in Figs. 1a–3a, in the range of A_p values from 1.18 through 0.27, clearly show that the evolution of the drag coefficient beyond the period of constant acceleration, i.e., for $S/R > (S/R)_v$, does not measurably depend on A_p; a drag overshoot occurs between $S/R \approx 4$ and $S/R = 5$ (at a relative distance covered in approximately half the vortex shedding period); the maximum C_d reaches a value of about 1.6 in the range of Reynolds numbers encountered; the drag overshoot occurs before the vortices exhibit appreciable asymmetry; and the first vortex does not shed for S/R smaller than about 9. In other words, the flow may be considered as impulsively started for all values of A_p larger than about 0.27.

The drag overshoot and the subsequent rapid decrease of C_d to a value slightly larger than its commonly accepted quasisteady value (at the corresponding Reynolds numbers) is related to but not caused by the onset of wake asymmetry, as manifested by the evolution of the lift force. Sarpkaya and Shoaffs[6] numerical simulations through the use of the vortex-element method[9] yielded a drag overshoot of about 1.6 at $S/R \approx 4$. For an impulsively started flow, with a Reynolds number of $Re = 1200$, Rumsey's[5] calculations yielded a minimum C_d at approximately $S/R \approx 1.6$ and a local maximum of $C_d \approx 1.44$ at about $S/R \approx 3.2$. Subsequently, C_d decreased gradually to about 0.7 over a range of S/R values from 3.2 to

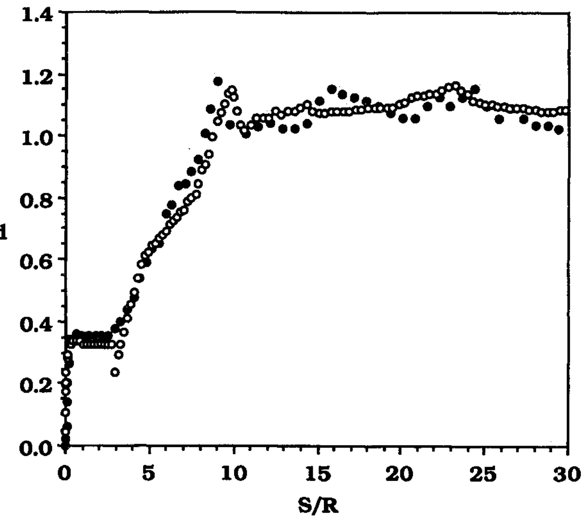

Fig. 6a Drag coefficient vs relative displacement for $A_p = 0.10$ (open circles) and for $A_p = 0.11$ (full circles).

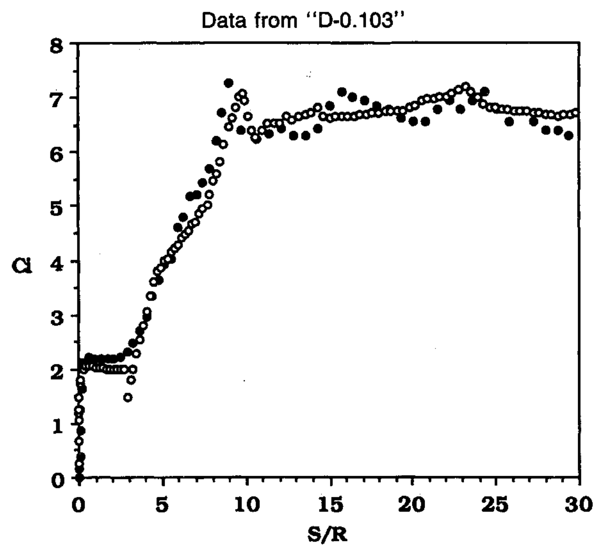

Fig. 6b Force coefficient C_i vs relative displacement for $A_p = 0.10$ (open circles) and for $A_p = 0.11$ (full circles).

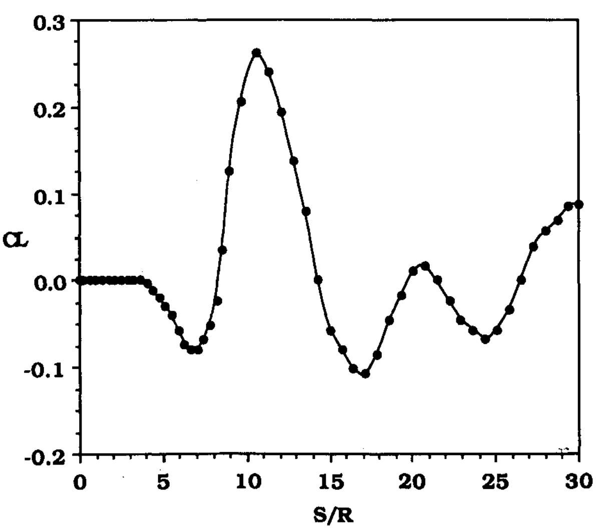

Fig. 6c Lift coefficient vs relative displacement for $A_p = 0.11$.

24. Loc's[1] computations yielded a local maximum drag coefficient of about 1.2 at $S/R \approx 4$ for a Reynolds number of $Re = 1000$. His calculations with different grid systems and time steps for $Re = 550$ have shown that S/R, at which the drag overshoot occurs, is somewhat larger for coarser grids. Loc has also shown that the evolution with time of the vorticity distribution at the cylinder surface for $Re = 1000$ changes very little for S/R larger than about 4.

Thoman and Szewczyk,[18] among the first to calculate numerically the flow about a cylinder, found for $Re = 4 \times 10^4$ that C_d is very high at the impulsive start of flow, decays rapidly to a minimum that decreases with increasing Reynolds number, and then rises to a peak value of about 1.5 at $S/R \approx 4$. The drag then decreases to its quasisteady value at about $S/R \approx 9$ and remains nearly constant thereafter. This is rather remarkable in view of the fact that the steady-state value of the drag coefficient is reached even before a single vortex is shed! Thoman and Szewczyk introduced an asymmetric disturbance, in the form of a 11.5-deg rotation of the cylinder at a constant velocity, at $S/R = 10$ and removed it at $S/R = 12$. This small disturbance triggered the start of vortex shedding if the Reynolds number is high enough to produce amplification, otherwise, the asymmetry produced by the artificial excitation decayed rapidly after $S/R = 12$. The separation points reached their quasisteady positions at about $S/R \approx 4$ for $Re = 4 \times 10^4$. Likewise, the vorticity distribution about the cylinder showed only minor changes (near the base of the cylinder) for S/R larger than about 5. Nevertheless, no attempt was made to explain the rapid drop in C_d following the drag overshoot at $S/R \approx 4$.

There may not be a simple explanation for the local maximum drag coefficient for a number of reasons. During the initial and symmetric development of vortices (for S/R smaller than about 4), the vorticity rapidly accumulates in the wake. The symmetric vortices grow to sizes considerably larger than those that would be found in the later stages of the motion where the vortices shed alternatingly. However, the accumulation of vorticity and the reduction of the backpressure are not sufficient to explain the drag overshoot. The size of the wake region at $S/R \approx 4$ is about one half of that at $S/R \approx 7$ (see, e.g., Rumsey[5]). Thus, simple vorticity accumulation is not sufficient for the C_d overshoot (note that for S/R larger than about 5 the vortex center does not change appreciably). The growth of the symmetric or nearly symmetric vortices (depending on the existing natural disturbances) is accompanied by other events. The so-called α-phenomenon (Bouard and Coutanceau[10]) disappears between $S/R \approx 4.8$ and 6.2, according to Rumsey's[5] calculations at $Re = 1200$ (the range of S/R values, over which the said phenomenon disappears, depends on Re). In fact, the pressure distribution in the vicinity of the secondary vortices changes accordingly. Finally, during the early stages of the motion the vorticity flux is expected to be considerably larger than the cross-wake transfer of oppositely signed vorticity (in fact, this is one of the reasons for the rapid accumulation of vorticity). However, as the wake size grows and the vortices begin to exhibit asymmetry, as well as three dimensionality (see Figs. 2b, 3b, 5h, and 6c), the cross-wake vorticity transfer is expected to increase significantly. It is surmised that it is the combination of the foregoing events that leads to the rapid rise and fall of the the drag coefficient at or near $S/R \approx 4.5$.

For A_p smaller than about 0.27, i.e., when the end of the acceleration period nearly coincides with or exceeds the S/R value at which the drag overshoot for impulsively-started flow occurs normally, the stabilizing effects of the acceleration help to maintain the wake symmetrical for longer periods of time (see Fig. 5b). The drag overshoot still occurs at or near $S/R \approx 4.5-5$, but the magnitude of maximum C_d is now somewhat smaller (about 1.3). Part of the reason for this is that the vorticity flux is expected to be smaller for a flow accelerating uniformly up to the time of drag overshoot relative to that for an impulsively started flow. The differences between the two cases may also be gleaned from a comparison of the distance covered prior to the onset of separation at the rear stagnation point. For an impulsively started flow about a circular cylinder, the said distance is $S/R = 0.351$, whereas for a uniformly accelerating flow it is $S/R = 0.52$ (see, e.g., Schlichting[20]). The corresponding distances for the spontaneous breaking away of the unsteady flow from the surface are $S/R \approx 1.35$ for the impulsive case and $S/R \approx 2.45$ for the constant-acceleration case (a more detailed discussion is given in Sarpkaya[19]).

Figures 6a and 6b show C_d and C_i for $A_p = 0.10$ and 0.11, [corresponding to $(S/R)_v = 10$ and 9.1], and Fig. 6c shows C_L for $A_p = 0.11$, all for $Re \approx 46{,}000$. The drag coeffocient rises sharply to a value of $C_{din} = \pi A_p$ immediately after the start of motion and remains nearly constant for a distance of about $\Delta(S/R) = 3$. As noted earlier, the rear separation for this flow begins at $S/R = 0.52$. The data shown in Fig. 6a, and other data not presented herein because of space limitations, show that the effect of the symmetrically growing small wake during a displacement of about $\Delta(S/R) \approx 3$ on the total drag is negligible relative to that of the inertial force. Figure 6h shows that the initial value of C_i is slightly larger than its ideal value of 2, partly due to the small, velocity-square-dependent form and skin-friction drags and partly due to the inevitable experimental errors. As the flow continues to accelerate, C_d begins to rise almost linearly with S/R, under the increasing dominance of the form drag over the inertial force, until the acceleration is removed or the boundary layer undergoes a transition. In Fig. 6a the acceleration is removed rapidly at $S/R = 9.1$ and 10, respectively, for the two cases shown. Clearly, C_d assumes its quasisteady value almost immediately. The inflection in the rise of C_d, between $S/R \approx 5$ and $S/R \approx 8$, is due to the asymmetrical growth and shedding of the first vortex (see Fig. 6c). The secondary oscillations in C_d beyond $S/R \approx 8$ are associated, as before, with the shedding of the additional vortices.

Concluding Remarks

In the data discussed in the foregoing the drag force for a given acceleration history was repeatable with minor variations (about ±5%). Extreme care was taken to ensure that the rate of deceleration at the end of the acceleration period was as high as possible and that there were no residual accelerations or decelerations. As noted in the Introduction, the effect of (d^2U/dt^2) and of the effect of residual decelerations can be very strong on the evolution of the drag force because the ratio of the inertial force to the drag force is proportional to $(\pi/2)A_p$ times the ratio of the inertia coefficient to the drag coefficient,[17] which, in turn, depends on A_p, Re, and S/R. However, as far as the repeatability of the lift force is concerned, it becomes clear very quickly that the inception and evolution of the lift force are sensitive to unknown and unknowable initial conditions, as previously noted by Sarpkaya.[12]

It is not generally appreciated that in physical experiments the symmetry is destroyed by the presence of multiple perturbation sources and that the numerical experiments cannot model the perturbations of nature yielding the final asymmetry. The initial conditions are never given and are never the same. As noted by Braza et al.,[21] "It is beyond the experimental ability of the researcher to control or even recognize all possible contingencies that may arise in an experiment designed to study the evolution of the asymmetry. When the flow is in a critical state to become asymmetrical, a very small cause can have a very great effect." This does not settle the question of when the first von Kármán vortex will shed from a circular cylinder. The fact that the "artificial disturbances" of short duration and permanent random perturbations lead to the same flow pattern indicates that "the periodic character of the flow appearing beyond a critical value of the Reynolds number is an intrinsic property of the Navier-Stokes equations and does not depend on the nature of the perturbations."[21] More-over, the perturbations seem to be responsible for only the change of the regime from steady to periodic flow, but they are not necessary as a source of energy to sustain the periodic flow Finally, it is rather remarkable (and somewhat fortunate) that the effect of the variability of the lift force in the evolution of the drag force is relatively small, even during the shedding of the first two vortices.

In summary, 1) the relative distance S/R at which the local maximum drag coefficient occurs does not depend on the acceleration parameter A_p for A_p larger than about 0.27 (corresponding to an almost impulsively started flow beyond the period of initial acceleration) and agrees reasonably well with the previous numerical predictions,[3,6,18] based on impulsive start ($A_p = \infty$). 2) For accelerations that end near $S/R \approx 5$ (e.g., for $A_p = 0.2$), a smaller drag maximum occurs. 3) For A_p smaller than about 0.10 or 0.15, the drag coefficient remains nearly constant for S/R smaller than about 3, and the drag force is almost entirely due to inertial effects (added mass plus the pressure gradient to accelerate the ambient flow, i.e., $C_i \approx 2$). For $S/R > 2$, C_d rises almost linearly while exhibiting a small inflection due to the shedding of the very first vortex. The rise in C_d is halted either by the termination of the acceleration or by transition in the boundary layers (not discussed further in this paper). There is no drag overshoot, and C_d reaches its quasisteady value with usual lift-induced oscillations. 4) The inception of the transverse force (lift force) depends on the initial disturbances. The lift force begins to develop for S/R larger than about 2, depending on A_p and, probably, on *Re*. 5) Impulsively started steady flow is a very special case of unsteady flows starting from rest and reaching a steady state over a time period. The numerical methods and codes should be applied to this type of asymmetric unsteady flows. Such studies will help to resolve the consequences of a specific unsteady input of given type and duration (e.g., acceleration or deceleration or their combinations) on the subsequent stages of the motion of a viscous fluid. Furthermore, the simulations will provide an excellent opportunity to discover many interesting and new vortex patterns, in addition to the well-known α and β phenomena,[10] whose role on the understanding of time-dependent separation and resistance are worthy of study in their own right. It is also extremely desirable that the numerical calculations be carried out to S/R values considerably greater than 6 or so without imposing symmetry.

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